



Answer the following questions

Question 1 :

- (a) Solve the initial value problem by using Laplace transform:

$$y'' + \lambda^2 y = \cos \lambda t \quad y(0) = 1, \quad y\left(\frac{\pi}{2\lambda}\right) = 1$$

Also solve this problem , if the boundary is given as $y(0) = 1, \quad y'\left(\frac{\pi}{\lambda}\right) = 1$

- (b) Draw and compute the Fourier series of the function:

$$f(t) = \begin{cases} t & 0 < x < 1 \\ 1-t & 1 < x < 2 \end{cases}$$

- (c) Test the convergence of the following series:

$$i) \sum_{n=1}^{\infty} \frac{n(x-1)^n}{2^n(3n-1)}$$

$$ii) \sum_{n=1}^{\infty} \frac{e^{\tan^{-1} n}}{n^2 + 1}$$

- (d) Evaluate the following integrals :

$$i) \iint_D \frac{1}{x} \cos \frac{y}{x} dA \quad \text{where } D = \left\{ (x, y) \mid \frac{\pi}{2} \leq x \leq \pi, 0 \leq y \leq x^2 \right\}$$

$$ii) \iiint_D (xz + yz) dV \quad \text{where } D = \left\{ (x, y, z) \mid 0 \leq x \leq 2, 0 \leq y \leq 3, 0 \leq z \leq -x + 4 \right\}$$

- (e) Find the Laplace transform of the functions:

$$i) f(t) = (e^{-2t} - 1)^2 \sin^2 t + u(t-2) \cosh 2(t-2) \quad ii) f(t) = e^{2t} \frac{\cos 3t}{t}$$

- (f) Find the mass and center of mass of the lamina that occupies the region D bounded by the curves $y = x^2$, $x = y^2$ and has density function $\rho(x, y) = \sqrt{x}$.

Question 2

(a) Find the inverse Laplace transform of the functions:

$$i) F(s) = \frac{2s^2}{(s^2 + 1)(s - 1)^2}$$

$$ii) F(s) = \frac{1}{2} \ln\left(1 + \frac{4}{s^2}\right)$$

(b) Find the inverse Laplace transform by using the convolution theorem

$$F(s) = \frac{1}{s^2(s^2 + 1)}$$

(c) Graph the function and then find its Laplace transform

$$f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$$

(d) Find the orthogonal trajectories of the following curve:

$$x^2 + y^2 + 2fy + 1 = 0, \quad \text{where } f \text{ is the parameter}$$

(e) Solve the following ordinary differential equations:

$$i) (D^2 + 2D + 2)y = 2e^{-x} \sin x$$

$$ii) (x^2 - 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$$

$$iii) \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

$$iv) y = 2px + y^2 p^3$$

$$v) 3e^x \tan y + (1 - e^x) \sec^2 x \frac{dy}{dx} = 0$$

$$vi) x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(f) Solve the system of simultaneous differential equations:-

$$\frac{dx}{dt} + 7x - y = e^{-t}$$

$$\frac{dy}{dt} + 2x + 5y = 0$$

With my best wishes

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This exam measures the following ILOs											
Question Number	Q 1-a	Q 1-a	Q 1-b	Q 5-c	Q4-c	Q5-b			Q3-b	Q 5-a	
	Q 2-a	Q 2-b	Q 2-c	Q3-c	Q3-a	Q4-b			Q4-a		
Skills	Knowledge & understanding Skills				Intellectual Skills				Professional Skills		